September 2007 S M Houghton

MATH3501 Modelling with Fluids

Example sheet 1

1. Find the general solution to these differential equations.
   (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^y x^2$
   (b) $x\frac{\mathrm{d}y}{\mathrm{d}x} + y = x^2$
   (c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = x$
   (d) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 \sin t$
   (e) $\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^t - 5y$
   (f) $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 1 + x$

2. Find the general solution for these coupled equations where x, y and z are all functions of t.
   (a) $\frac{\mathrm{d}x}{\mathrm{d}t} = 1$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 3 + y$, $\frac{\mathrm{d}z}{\mathrm{d}t} = (z+2)\,x$.
   (b) $\frac{\mathrm{d}x}{\mathrm{d}t} = 2x + t\mathrm{e}^{2t}$, $\frac{\mathrm{d}y}{\mathrm{d}t} = z$, $\frac{\mathrm{d}z}{\mathrm{d}t} = z$.

3. Find the general solution for these coupled equations where x, y and z are all functions of t.
   (a) $\frac{\mathrm{d}x}{\mathrm{d}t} = y$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 4\,(x-1)$.
   (b) $\frac{\mathrm{d}x}{\mathrm{d}t} = y$, $\frac{\mathrm{d}y}{\mathrm{d}t} = x + 1$, $\frac{\mathrm{d}z}{\mathrm{d}t} = x$.

4. A two-dimensional velocity field is given by $\mathbf{u} = (yt, 1)$. Find the equations for the streamlines, pathlines and streaklines. Sketch the streamline passing through point $(1,1)$ at $t = 0$, also the particle path of a particle released from point $(1,1)$ at $t = 0$ and the streakline formed at $t = 0$ by dye released from $(1,1)$.

5. Repeat question 4 with the velocity field $\mathbf{u} = (x, -y)$. Why do the three visualization methods give the same result in this case?

6. A steady two-dimensional flow is given by $\mathbf{u} = (\alpha x, -\alpha y)$, where α is a constant.
   (a) Find the equation for a general streamline in the flow. Sketch some of the streamlines.
   (b) At time $t = 0$ fluid on the curve $x^2 + y^2 = a^2$ is marked with dye. Find the equation for this material fluid curve for $t > 0$.
   (c) Does the area inside the curve change with time? If not, why?

7. Repeat question 6, but with the shear flow $\mathbf{u} = (\gamma y, 0)$.

Please send any comments, or corrections, to S M Houghton.
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